

Abstract Algebra II

Exercise Set 1: Solvability

Commutant

Question 1. Verify that the commutant of a group $K(G)$ is a subgroup of G . Verify that $K(G) = \{e\}$ if and only if G is commutative.

Question 2. Find the commutant of the following groups:

- $\mathbb{Z}_5$;
- $\mathbb{Z}_2 \times \mathbb{Z}_2$ (the Klein-4 group, V_4);
- the group of symmetric transformations of a regular pentagon;
- the group of *rotations* of a regular tetrahedron;
- S_5 ;
- $\text{SL}_2(\mathbb{F}_2)$, the group 2×2 matrices with entries from the field of order 2 (simple $\mathbb{Z}_2$ with multiplication) and determinant 1.

Soluble groups

Question 3. Write the chain of commutant subgroups for each of the following groups. In each case, use this to decide whether the groups are soluble:

- $\mathbb{Z}_5$
- $\mathbb{Z}_2 \times \mathbb{Z}_2$ (the Klein-4 group, V_4);
- the group of symmetric transformations of a regular pentagon;
- the group of *rotations* of a regular tetrahedron
- S_5 ;
- $A_5 \times C_2$